Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Wednesday 10 June 2020

Afternoon (Time: 2 hours 30 minutes) | Paper Reference **WMA02/01**

Mathematics

International Advanced Level
Core Mathematics C34

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

Pearson

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1. The curve C has equation

$$y = \frac{16}{3(5x-2)^3} \qquad x > \frac{2}{5}$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(2)

The point P with x coordinate $\frac{4}{5}$ lies on C.

(b) Find the equation of the tangent to C at P writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

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Question 1 continued

Q1

(Total 6 marks)

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2. The growth of pond weed on the surface of a pond is being investigated.

The surface area of the pond covered by the weed, $A\,\text{m}^2$, is modelled by the equation

$$A = \frac{1200e^{0.04t}}{4e^{0.04t} + 1} \qquad t \in \mathbb{R}, t \geqslant 0$$

where t is the number of weeks after the start of the investigation.

Using the model,

(a) calculate the surface area of the pond covered by the weed at the start of the investigation,

(1)

(b) calculate the value of t when $A = 260$, giving your answer to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

The pond weed continues to grow until it completely covers the surface of the pond.

Using the model,

(c) deduce the maximum possible surface area of the pond.

(1)

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Question 2 continued

Q2

(Total 6 marks)

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3.

$$f(x) = \frac{2x^2 + 21}{(1 - 4x)(3 + x)^2} \qquad |x| < \frac{1}{4}$$

Given that

$$f(x) = \frac{A}{(1 - 4x)} + \frac{B}{(3 + x)^2} + \frac{C}{(3 + x)}$$

where A, B and C are constants,

(a) (i) find the value of A and the value of B,

(ii) show that $C = 0$

(4)

(b) Hence, or otherwise, find the series expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. Give each coefficient as a fully simplified fraction.

(6)

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Question 3 continued

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Question 3 continued

Q3

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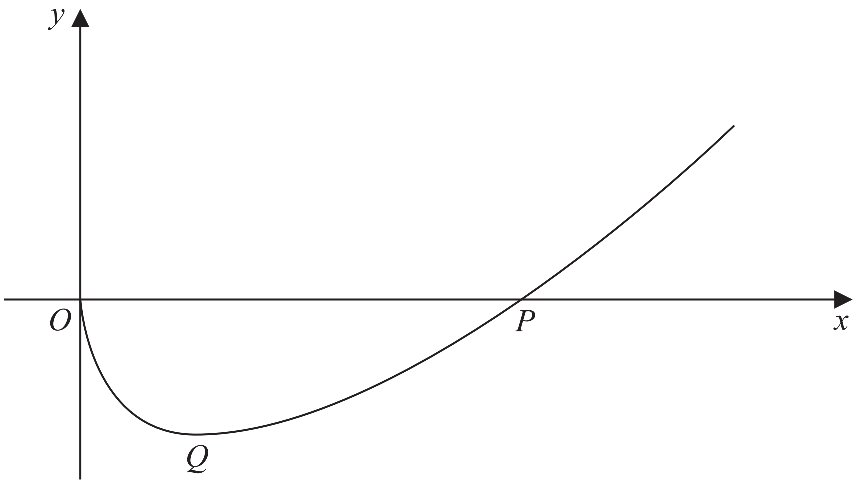

Figure 1

Figure 1 shows a sketch of the curve with equation

$$y = x\ln x - 6\sqrt{x}, \quad x > 0$$

The curve crosses the x-axis at the point P and has a minimum turning point at Q.

(a) Show that the x coordinate of P lies in the interval [8, 8.5].

(2)

(b) Show that the x coordinate of Q is a solution of the equation

$$x = e^{\frac{3}{\sqrt{x}} - 1}$$

(4)

Using the iterative formula

$$x_{n+1} = e^{\frac{3}{\sqrt{x_n}} - 1} \qquad \text{with } x_1 = 2.5$$

(c) find the value of x_2 and the value of x_3 to 3 decimal places.

(2)

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

(Total 8 marks)

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5. The functions f and g are defined by

$$f(x) = \frac{3}{x} - 1 \qquad x \in \mathbb{R}, x \neq 0$$

$$g(x) = e^{4x} \qquad x \in \mathbb{R}$$

(a) Find $f(x + 2) - 2f(x)$

Write your answer as a single fraction in simplest form.

(3)

(b) Find $f^{-1}(7)$

(2)

(c) Find the exact solution to the equation

$$fg(x) = 8$$

(3)

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Question 5 continued

Q5

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6. A curve has equation

$$2y^3 + 3x^2y - 3x + 2 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y, fully simplifying your answer.

(5)

The curve cuts the x-axis at the point P.

The normal to the curve at P meets the y-axis at the point Q.

Given that O is the origin,

(b) find, in simplest form, the exact area of triangle OPQ.

(4)

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Question 6 continued

Q6

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7. (a) Use the identity for $\tan(A + B)$ to show that

$$\tan 3A \equiv \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

(4)

(b) Hence solve, for $-\frac{\pi}{6} < x < \frac{\pi}{6}$

$$\tan 3x = 4\tan x$$

Give each answer to 3 significant figures where appropriate.

(4)

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Question 7 continued

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Question 7 continued

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Q7

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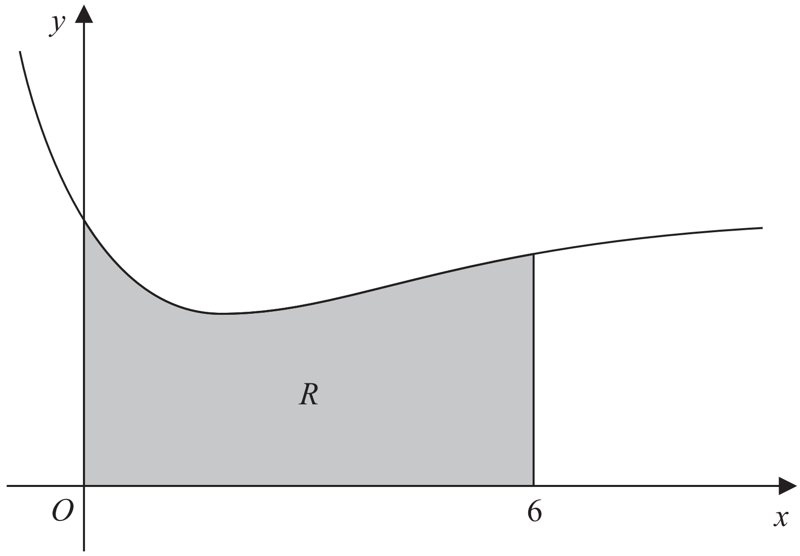

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 4 - 2xe^{-\frac{1}{2}x} \qquad x \in \mathbb{R}$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = 6$

The table below shows corresponding values of x and y for $y = 4 - 2xe^{-\frac{1}{2}x}$ with the values of y given to 4 decimal places where appropriate.

x	0	1.5	3	4.5	6
y	4	2.5829	2.6612	3.0514	3.4026

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for the area of R, giving your answer to 3 significant figures.

(3)

(b) Use calculus to find, in simplest form, the exact area of R.

(6)

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Question 8 continued

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Q8

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9.

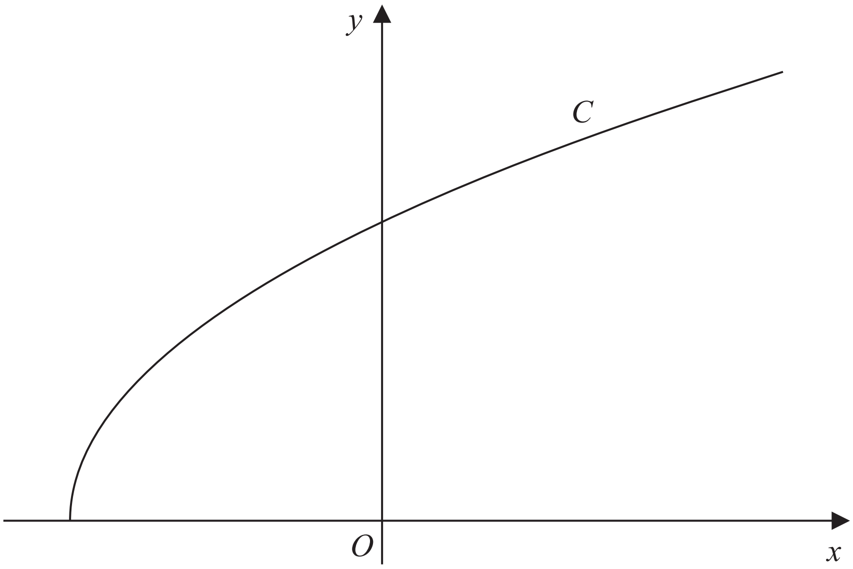

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 1 - 8\cos 2t \qquad y = 9\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

(a) Use parametric differentiation to find the value of $\frac{dy}{dx}$ at the point on C where $x = 5$

Give your answer in the form $k\sqrt{3}$, where k is a constant to be found.

(4)

(b) Show that all points on C satisfy

$$y = \frac{9}{4}\sqrt{x+7}$$

(3)

The curve C has equation $y = f(x)$ where f is the function

$$f(x) = \frac{9}{4}\sqrt{x+7} \qquad a \leqslant x \leqslant b$$

and a and b are constants.

(c) Find the value of a and the value of b.

(2)

(d) State the range of f.

(1)

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Question 9 continued

Q9

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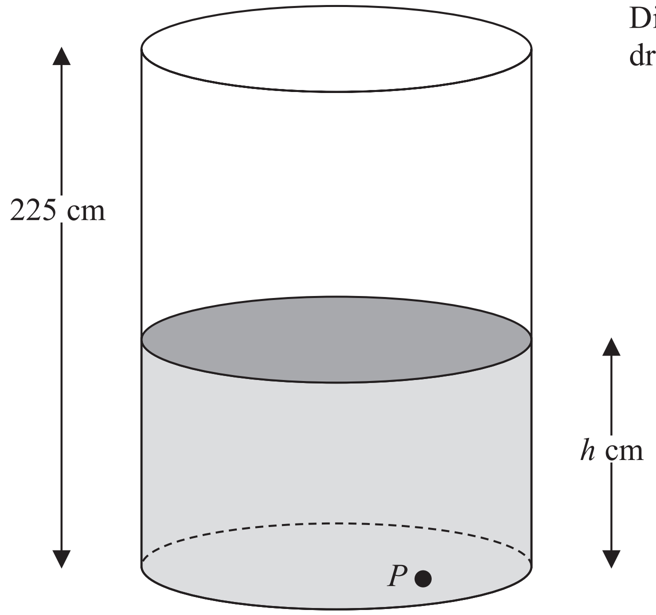

Figure 4

Figure 4 shows a vertical cylindrical tank of height 225 cm containing a liquid.
The liquid is leaking out of a hole P in the base of the tank.

At time t minutes after the leaking starts, the height of the liquid in the tank is h cm.

The rate at which the height of the liquid in the tank is decreasing, at any time t minutes, is modelled as being proportional to the square root of the height of the liquid in the tank.

When $t = 0$, $h = 225$ and when $t = 125$, $h = 100$

The liquid stops leaking from the tank when $h = 0$

By forming and solving a differential equation,

(a) show that the model leads to the equation

$$h = (15 - 0.04t)^2 \qquad 0 \leqslant t \leqslant a$$

stating the value of the constant a.

(7)

(b) Find, according to the model, the time taken for the height of the liquid in the tank to decrease from 100 cm to 50 cm. Give your answer to the nearest minute.

(3)

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Q10

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11.

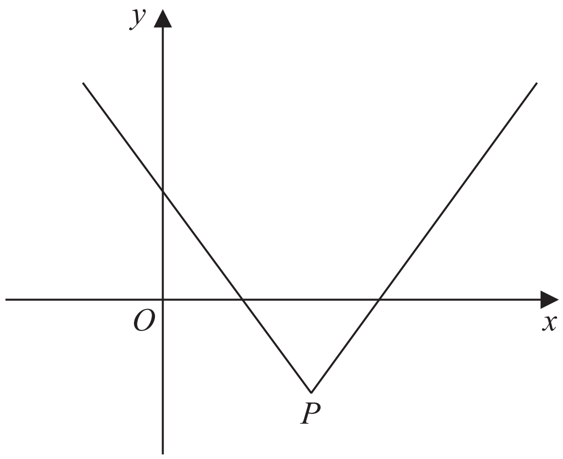

Figure 5

Figure 5 shows a sketch of part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = |x - 2a| - 3b, \qquad x \in \mathbb{R}$$

and where a and b are positive constants.

All answers to parts (a), (b), (c) and (d) should be expressed in terms of *a* and/or *b*.

(a) Find the values of x such that $\mathrm{f}(x) = 0$

(2)

The point P, as shown in Figure 5, is the vertex of the graph.

(b) State the coordinates of the point P.

(1)

(c) State the coordinates of the image of P under the transformation represented by the graph with equation

(i) $y = 2|\mathrm{f}(x)|$

(ii) $y = 3\mathrm{f}(2x)$

(2)

(d) Solve the equation

$$|x - 2a| - 3b = 2x + a$$

(3)

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Question 11 continued

Q11

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12. (i) Find

$$\int \frac{4}{(5y-7)^4}\,dy$$

(2)

(ii) Find, in simplest form,

$$\int (1-4\tan 3x)^2\,dx$$

(4)

(iii) Using the substitution $u = 1 + 2\cos\theta$, or otherwise, find

$$\int_0^{\frac{\pi}{2}} \frac{2\sin 2\theta}{1+2\cos\theta}\,d\theta$$

giving your answer in the form $\ln(Ae^2)$, where A is a constant to be found.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

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Question 12 continued

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Question 12 continued

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Question 12 continued

Q12

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13. Ellen bungee jumps from a high platform.

Ellen's distance above the ground, H metres, is modelled by the equation

$$H = 60 + \frac{50\cos(0.5t)}{e^{0.2t}} \qquad t \in \mathbb{R}, t \geqslant 0$$

where t is the time measured in seconds from when she jumps from the platform.

(a) (i) Find, in simplest form, $\dfrac{dH}{dt}$

(ii) Hence show that when $\dfrac{dH}{dt} = 0$, the values of t satisfy the equation

$$\tan(0.5t) = -0.4$$

(5)

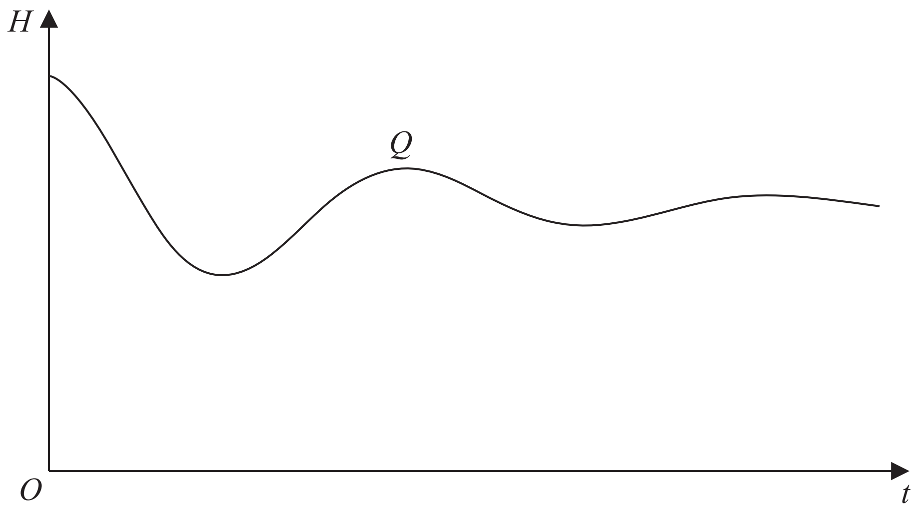

Figure 6

Figure 6 shows a sketch of H against t. The point Q, shown in Figure 6, represents the greatest distance above the ground to which Ellen bounces after jumping from the platform.

Using the answer to (a)(ii),

(b) find the value of t and the value of H at the point Q, giving your answers to 3 significant figures.

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Q13

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14. The point P with coordinates (3, 2, 9) lies on the line l_1
The point Q also lies on l_1

Given that $\overrightarrow{PQ} = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$

(a) find the coordinates of Q.
(1)

The point A has coordinates (8, 2, 6)

(b) Find the size of the angle APQ, giving your answer in degrees to 2 decimal places.
(3)

The line l_2 passes through the point A and is parallel to the line l_1

(c) Find a vector equation for l_2
(2)

The point X lies on l_1 and the point X is distinct from the point Q.

Given that the area of triangle APQ is the same as the area of triangle APX,

(d) find the coordinates of X.
(2)

The point Y lies on l_2

Given that PY is perpendicular to l_2

(e) find the coordinates of Y.
(5)

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Question 14 continued

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Q14

TOTAL FOR PAPER: 125 MARKS

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